

Pion dispersion relation at finite temperature in the linear sigma model from chiral Ward identities

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We develop one-loop effective vertices and propagators in the linear sigma model at finite temperature satisfying the chiral Ward identities. We use these in turn to compute the pion dispersion relation in a pion medium for small momentum and temperatures on the order of the pion mass at next to leading order in the parameter $m_\pi^2/4\pi^2 f_\pi^2$ and to zeroth order in the parameter $(m_\pi/m_\sigma)^2$. We show that this expansion reproduces the result obtained from chiral perturbation theory at leading order. The main effect is a perturbative, temperature-dependent increase of the pion mass. We find no qualitative changes in the pion dispersion curve with respect to the leading order behavior in this kinematical regime.

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The propagation properties of pions in a thermal hadronic environment are a key ingredient for the proper understanding of several physical processes taking place in dense and hot plasmas. The scenarios include dense astrophysical objects such as neutron stars –where the inclusion of mesonic degrees of freedom is essential to determine the equation of state of matter in the inner core– and the evolution of the highly interacting region formed in the aftermath of a high-energy heavy-ion collision. In the latter context, knowledge of the pion dispersion relation would help to properly account for the component of the dilepton spectrum originated in the hadronic phase, below the critical temperatures and densities for the formation of a quark-gluon plasma (QGP) [1], and to shed light on possible collective surface phenomena which are speculated to occur as a consequence of the mainly attractive interaction between pions and nuclei [2].

In order to account for the hadronic degrees of freedom at temperatures and densities below the QCD phase transition to a QGP, one needs to resort to effective chiral theories whose basic ingredient is the fact that pions are Goldstone bosons, originated in the spontaneous breakdown of chiral symmetry. Chiral perturbation theory (χ PT) is one of such effective theories that has been successfully employed to show the well known result that at leading perturbative order and at low momentum, the modification of the pion dispersion curve in a pion medium is just a constant, temperature dependent, increase of the pion mass [3]. Any deviation from this leading order behavior in perturbation theory can only be looked for at higher orders. Second order calculations of the pion dispersion curve can be found in Refs. [4,5].

The simplest realization of chiral symmetry is nevertheless provided by the much studied linear sigma model which possesses the convenient feature of being a renormalizable field theory, both at zero [6] and at finite temperature [7]. This model has lately received a boost of attention in view of recent theoretical

results [8] and analyses of data [9] that seem to confirm, though not without controversy [10], that a broad scalar resonance, with a mass in the vicinity of 600 MeV –that can be identified with the σ -meson– indeed exists.

In addition to the above mentioned characteristics, the linear sigma model also reproduces –as we will show– the leading order modification to the pion mass in a thermal pion medium (for temperatures on the order of the pion mass), when use is made of a systematic expansion in the ratio $(m_\pi/m_\sigma)^2$ at zeroth order, where m_σ , m_π are the vacuum sigma and pion masses, respectively. This important point, often missed in the literature [11], provides the connection between the linear sigma model and χ PT at finite temperature.

In this work, we use the linear sigma model to construct effective vertices and propagators to one loop. We use these in turn to compute the one loop modification to the pion propagator in a pion medium. We require that the effective vertices and propagators thus constructed satisfy the chiral Ward identities [12]. The net result is a next to leading order calculation in the parameter $m_\pi^2/4\pi^2 f_\pi^2$, where f_π is the pion decay constant and to zeroth order in the parameter $(m_\pi/m_\sigma)^2$. From the effective pion propagator, we explore the behavior of the pion dispersion curve at small momentum and for temperatures on the order of the pion mass. Though the momentum dependence of the correction term is non-trivial, we find no qualitative difference from the leading order result in this regime. Possible effects due to a high nucleonic density as well as general details of the calculation are left out for a future work.

The Lagrangian for the linear sigma model, including only the mesonic degrees of freedom and after the explicit inclusion of the chiral symmetry breaking term, can be expressed as [6]

$$\mathcal{L} = \frac{1}{2} [(\partial\pi)^2 + (\partial\sigma)^2 - m_\pi^2\pi^2 - m_\sigma^2\sigma^2] - \lambda^2 f_\pi \sigma (\sigma^2 + \pi^2) - \frac{\lambda^2}{4} (\sigma^2 + \pi^2)^2, \quad (1)$$

where π and σ are the pion and sigma fields, respectively, and the coupling λ^2 is given by

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}. \quad (2)$$

From the Lagrangian in Eq. (1), one obtains the Green's functions and the Feynman rules to be used in perturbative calculations, in the usual manner. Thus, in particular, the bare pion and sigma propagators $\Delta_\pi(P)$, $\Delta_\sigma(Q)$ and the bare one-sigma two-pion and four-pion vertices Γ_{12}^{ij} , Γ_{04}^{ijkl} are given by (hereafter, capital letters are used to denote four momenta)

$$\begin{aligned} i\Delta_\pi(P)\delta^{ij} &= \frac{i}{P^2 - m_\pi^2} \delta^{ij} \\ i\Delta_\sigma(Q) &= \frac{i}{Q^2 - m_\sigma^2} \\ i\Gamma_{12}^{ij} &= -2i\lambda^2 f_\pi \delta^{ij} \\ i\Gamma_{04}^{ijkl} &= -2i\lambda^2 (\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}). \end{aligned} \quad (3)$$

These Green's functions are sufficient to construct the modification to the pion propagator, both at zero and finite temperature, at any given perturbative order.

An alternative approach consists on exploiting the relations that chiral symmetry imposes among different n-point Green's functions. These relations, better known as chiral Ward identities (χ WIs), are a direct consequence of the fact that the divergence of the axial current may be used as an interpolating field for the pion. For example, two of the χ WIs satisfied –order by order in perturbation theory– by the functions $\Delta_\pi(P)$, $\Delta_\sigma(Q)$, Γ_{12}^{ij} and Γ_{04}^{ijkl} are [6]

$$f_\pi \Gamma_{04}^{ijkl}(\cdot; 0, P_1, P_2, P_3) = \Gamma_{12}^{kl}(P_1; P_2, P_3) \delta^{ij} + \Gamma_{12}^{lj}(P_2; P_3, P_1) \delta^{ik} + \Gamma_{12}^{jk}(P_3; P_1, P_2) \delta^{il}$$

$$f_\pi \Gamma_{12}^{ij}(Q; 0, P) = [\Delta_\sigma^{-1}(Q) - \Delta_\pi^{-1}(P)] \delta^{ij}, \quad (4)$$

where momentum conservation at the vertices is implied, that is $P_1 + P_2 + P_3 = 0$ and $Q + P = 0$. Therefore, any perturbative modification of one of these functions introduces modifications in other, when the former are related to the latter through χ WIs.

At one loop and after renormalization, the sigma propagator is modified by finite terms. At zero temperature this modification is purely imaginary and its physical origin is that a sigma particle, with a mass larger than twice the mass of the pion, is unstable and has a (large) non-vanishing width coming from its decay channel into two pions. At finite temperature the modification results on real and imaginary parts. The real part modifies the sigma dispersion curve whereas the imaginary part represents a temperature dependent contribution to the sigma width.

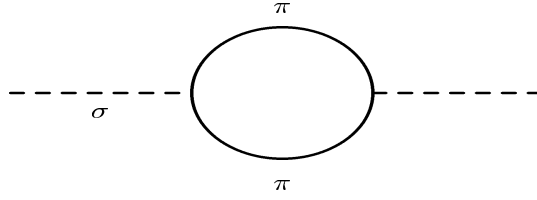


FIG. 1. Dominant contribution to the sigma self-energy at one loop.

After renormalization, the leading one-loop contribution to the sigma self-energy, in an expansion in the parameter $(m_\pi/m_\sigma)^2$, can be shown [13] to arise from the diagram depicted in Fig. 1. In the imaginary-time formalism of Thermal Field Theory (TFT), the expression for this diagram is given as $6\lambda^4 f_\pi^2 I(\omega, q)$, where the function I is defined by

$$I(\omega, q) = T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + k^2 + m_\pi^2} \frac{1}{(\omega_n - \omega)^2 + (\mathbf{k} - \mathbf{q})^2 + m_\pi^2}. \quad (5)$$

Here $\omega = 2m\pi T$ and $\omega_n = 2n\pi T$ (m, n integers) are discrete boson frequencies and $q = |\mathbf{q}|$. From Eq. (5) we obtain the time-ordered version I^t of the function I , after analytical continuation to Minkowski space. The imaginary part of I^t is given by

$$\text{Im} I^t(q_0, q) = \frac{\varepsilon(q_0)}{2i} [I(i\omega \rightarrow q_0 + i\epsilon, q) - I(i\omega \rightarrow q_0 - i\epsilon, q)]$$

$$= -\frac{1}{16\pi} \left\{ a(Q^2) + \frac{2T}{q} \ln \left(\frac{1 - e^{-\omega_+(q_0, q)/T}}{1 - e^{-\omega_-(q_0, q)/T}} \right) \right\} \Theta(Q^2 - 4m_\pi^2), \quad (6)$$

where $Q^2 = q_0^2 - q^2$, ε and Θ are the sign and step functions, respectively, and the functions a and ω_\pm are given by

$$a(Q^2) = \sqrt{1 - \frac{4m_\pi^2}{Q^2}}$$

$$\omega_\pm(q_0, q) = \frac{|q_0| \pm a(Q^2)q}{2}, \quad (7)$$

whereas the real part of I^t at $Q = 0$ is given by

$$\text{Re}I^t(0) = -\frac{1}{8\pi^2} \int_0^\infty \frac{dk}{E_k} [1 + 2f(E_k)] , \quad (8)$$

where $E_k = \sqrt{k^2 + m_\pi^2}$ and the function f is the Bose-Einstein distribution

$$f(E_k) = \frac{1}{e^{E_k/T} - 1} . \quad (9)$$

Therefore, the one-loop effective sigma propagator becomes

$$i\Delta_\sigma^*(Q) = \frac{i}{Q^2 - m_\sigma^2 + 6\lambda^4 f_\pi^2 I^t(Q)} . \quad (10)$$

The temperature-independent infinities are absorbed into the redefinition of the physical masses and coupling constants by the introduction of suitable counterterms in the usual manner.

In order to preserve the χ WIs expressed in Eq. (4), the corresponding one-loop effective one-sigma two-pion and four-pion vertices become

$$\begin{aligned} i\Gamma_{12}^{*ij}(Q; P_1, P_2) &= -2i\lambda^2 f_\pi \delta^{ij} [1 - 3\lambda^2 I^t(Q)] \\ i\Gamma_{04}^{*ijkl}(&; P_1, P_2, P_3, P_4) = -2i\lambda^2 \{ [1 - 3\lambda^2 I^t(P_1 + P_2)] \delta^{ij} \delta^{kl} \\ &+ [1 - 3\lambda^2 I^t(P_1 + P_3)] \delta^{ik} \delta^{jl} \\ &+ [1 - 3\lambda^2 I^t(P_1 + P_4)] \delta^{il} \delta^{jk} \} . \end{aligned} \quad (11)$$

It can be shown [13] that the functions in Eq. (11) arise from considering all of the possible one-loop contributions to the one-sigma two-pion and four-pion vertices, when maintaining only the zeroth order terms in a systematic expansion in the parameter $(m_\pi/m_\sigma)^2$.

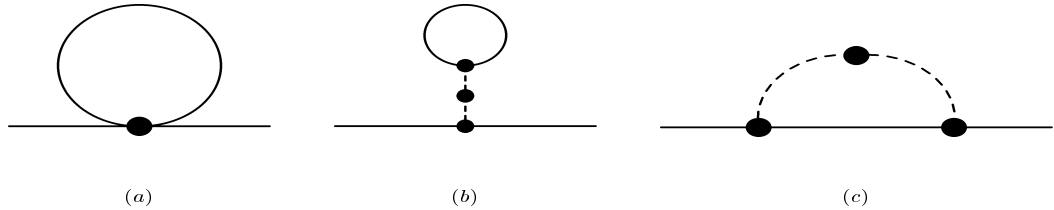


FIG. 2. Dominant contributions to the pion self-energy at one loop in the effective expansion. The heavy dots denote the effective vertices and propagators.

We now use the above effective vertices and propagator to construct the one-loop modification to the pion self-energy. The leading contributions come from the diagrams depicted in Fig. 2. These are, explicitly

$$\begin{aligned} \Pi_a(P) \delta^{ij} &= \lambda^2 \delta^{ij} \int_\beta \frac{d^4 K}{(2\pi)^4} \frac{1}{K^2 - m_\pi^2} \{5 - 9I^t(0) - 6\lambda^2 I^t(P + K)\} \\ \Pi_b(P) \delta^{ij} &= -\lambda^2 \delta^{ij} \left(\frac{6\lambda^2 f_\pi^2}{m_\sigma^2} \right) \int_\beta \frac{d^4 K}{(2\pi)^4} \frac{1}{K^2 - m_\pi^2} \left\{ \frac{[1 - 3\lambda^2 I^t(0)]^2}{[1 - \frac{6\lambda^4 f_\pi^2}{m_\sigma^2} I^t(0)]} \right\} \\ \Pi_c(P) \delta^{ij} &= -\lambda^2 \delta^{ij} \left(\frac{4\lambda^2 f_\pi^2}{m_\sigma^2} \right) \int_\beta \frac{d^4 K}{(2\pi)^4} \frac{1}{K^2 - m_\pi^2} \left\{ \frac{[1 - 3\lambda^2 I^t(P + K)]^2}{[1 - \frac{6\lambda^4 f_\pi^2}{m_\sigma^2} I^t(P + K) - \frac{(P+K)^2}{m_\sigma^2}]} \right\} , \end{aligned} \quad (12)$$

where the subindex β means that the integrals are to be computed at finite temperature. We now expand the denominators in the second and third of Eqs. (12), keeping only the leading order contribution when considering m_π , T and P as small compared to m_σ . Adding up the above three terms and to zeroth order in $(m_\pi/m_\sigma)^2$ where

$$\lambda^2 \left(1 - \frac{2\lambda^2 f_\pi^2}{m_\sigma^2} \right) \approx \frac{m_\pi^2}{2f_\pi^2}, \quad (13)$$

the pion self-energy can be written as

$$\begin{aligned} \Pi(P) = & \left(\frac{m_\pi^2}{2f_\pi^2} \right) T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{K^2 + m_\pi^2} \left\{ 5 + 2 \left(\frac{P^2 + K^2}{m_\pi^2} \right) \right. \\ & \left. - \left(\frac{m_\pi^2}{2f_\pi^2} \right) [9I^t(0) + 6I^t(P + K)] \right\}, \end{aligned} \quad (14)$$

where we carry out the calculation in the imaginary-time formalism of TFT with $K = (\omega_n, \mathbf{k})$ and $P = (\omega, \mathbf{p})$. The pion dispersion relation is thus obtained from the solution to

$$P^2 + m_\pi^2 + \text{Re}\Pi(P) = 0, \quad (15)$$

after the analytical continuation $i\omega \rightarrow p_o + i\epsilon$.

As it stands, Eq. (14) contains a temperature-dependent infinity coming from the product of the vacuum piece in $\text{Re}I^t(0)$ and the temperature-dependent piece in the indicated integral, as well as vacuum infinities. However, it is well known that finite temperature does not introduce new divergences and that whatever regularization and renormalization is needed at zero temperature, it will also be necessary and sufficient at finite temperature. Let us recall that at one loop, renormalization is carried out by introducing appropriate counterterms into the original Lagrangian and that at this level, no temperature-dependent infinities arise. Going to two loops, one will always encounter temperature-dependent infinities in integrals involving only the bare terms of the original Lagrangian. The remarkable property of renormalizable theories is that these temperature-dependent infinities are exactly canceled by the contribution from the loops computed by using the counterterms that were introduced at one loop. The above was explicitly shown for the case of the self-energy in the ϕ^4 theory by Kislinger and Morley [14] and for the sigma vacuum expectation value in the linear sigma model by Mohan [7]. For this reason, here we omit the renormalization procedure and consider only the finite, temperature-dependent terms throughout the rest of the calculation and refer the reader to the cited references for details.

Let us first look at the dispersion relation at leading order. After analytical continuation all the terms are real. The integrals involved are

$$\begin{aligned} T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{K^2 + m_\pi^2} & \rightarrow \frac{1}{2\pi^2} \int_0^\infty \frac{dk k^2}{E_k} f(E_k) \\ & \equiv \frac{m_\pi^2}{2\pi^2} g(T/m_\pi) \\ T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{K^2}{K^2 + m_\pi^2} & \rightarrow -m_\pi^2 \left(\frac{m_\pi^2}{2\pi^2} \right) g(T/m_\pi), \end{aligned} \quad (16)$$

where g is a dimensionless function of the ratio T/m_π and the arrows indicate only the temperature dependence of the expressions. Thus, the dispersion relation results from

$$[1 + 2 \xi g(T/m_\pi)] (p_0^2 - p^2) - [1 + 3 \xi g(T/m_\pi)] m_\pi^2 = 0, \quad (17)$$

with $\xi = m_\pi^2/4\pi^2 f_\pi^2 \ll 1$. For $T \sim m_\pi$, $g(T/m_\pi) \sim 1$, therefore, at leading order and in the kinematical regime that we are considering, Eq. (17) can be written as

$$p_0^2 = p^2 + m_\pi^2 [1 + \xi g(T/m_\pi)], \quad (18)$$

which coincides with the result obtained from χ PT [3].

We now look at the next to leading order terms in Eq. (14). The first of these is purely real and represents a constant, second order shift to the pion mass squared

$$- 9 \left(\frac{m_\pi^2}{2f_\pi^2} \right)^2 T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{I^t(0)}{K^2 + m_\pi^2} \rightarrow \frac{9}{2} \xi^2 g(T/m_\pi) h(T/m_\pi) m_\pi^2, \quad (19)$$

where h is a dimensionless function of the ratio T/m_π defined by

$$h(T/m_\pi) \equiv \int_0^\infty \frac{dk}{E_k} f(E_k). \quad (20)$$

The remaining term in Eq. (14) shows a non-trivial dependence on P . It involves the function S defined by

$$S(P) \equiv T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{I^t(P+K)}{K^2 + m_\pi^2}. \quad (21)$$

The sum is performed by resorting to the spectral representation of I^t and $(K^2 + m_\pi^2)^{-1}$. Thus, the real part of the retarded version of S , after analytical continuation is

$$\begin{aligned} \text{Re} S^r(p_0, p) &\equiv \frac{1}{2} [S(i\omega \rightarrow p_0 + i\epsilon, p) + S(i\omega \rightarrow p_0 - i\epsilon, p)] \\ &= -\mathcal{P} \int \frac{d^3 k}{(2\pi)^3} \int_{-\infty}^\infty \frac{dk_0}{2\pi} \int_{-\infty}^\infty \frac{dk'_0}{2\pi} [1 + f(k_0) + f(k'_0)] \\ &\quad \frac{2\pi \varepsilon(k'_0) \delta[k_0'^2 - (\mathbf{k} - \mathbf{p})^2 - m_\pi^2] 2 \text{Im} I^t(k_0, k)}{p_0 - k_0 - k'_0}, \end{aligned} \quad (22)$$

where \mathcal{P} represents the principal part of the integral.

The integration in Eq. (22) can only be performed numerically. Figure 3 shows a plot of the temperature-dependent terms of the function $\tilde{S}(\tilde{p}_0, p) = - (24\pi^4) \text{Re} S^r(\tilde{p}_0, p)$ for $T = m_\pi$ and \tilde{p}_0 taken as the solution of Eq. (15) up to the P -independent terms of Eq. (14). From this figure we see that \tilde{S} is non-vanishing at $p = 0$ and thus it also contributes to the perturbative increase of the pion mass. We also notice that \tilde{S} is a monotonically increasing function of p in the kinematical range considered. However, this increase is not strong enough to cause a qualitatively significant change on the shape of the dispersion curve, given that this function contributes multiplied by the perturbative parameter ξ^2 .

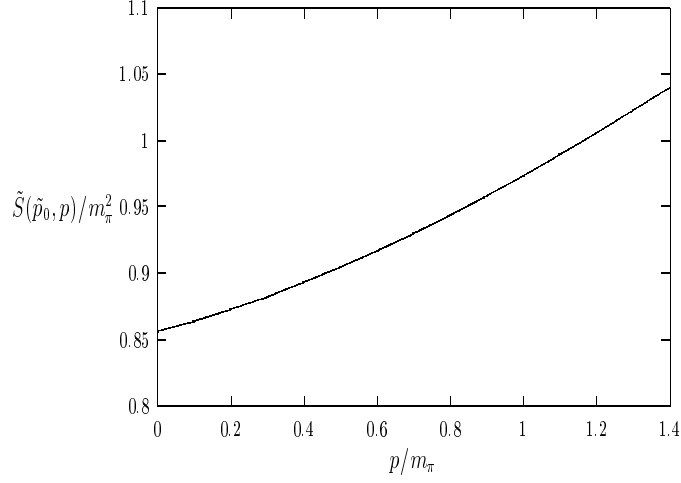


FIG. 3. $\tilde{S}(\tilde{p}_0, p)/m_\pi^2$ as a function of p/m_π for $T = m_\pi$. The function shows a mild monotonical increase in the kinematical region considered. This increase is not enough to change the qualitative behavior of the dispersion curve, given that \tilde{S} contributes at second order in the expansion parameter ξ .

Including all the terms, the dispersion relation up to next to leading order, for $T \sim m_\pi$ and in the small momentum region is obtained as the solution to

$$p_0^2 = p^2 + \left\{ 1 + \xi g(T/m_\pi) + \frac{\xi^2}{2} g(T/m_\pi) [9h(T/m_\pi) - 4g(T/m_\pi)] \right\} m_\pi^2 + \xi^2 \tilde{S}(p_0, p). \quad (23)$$

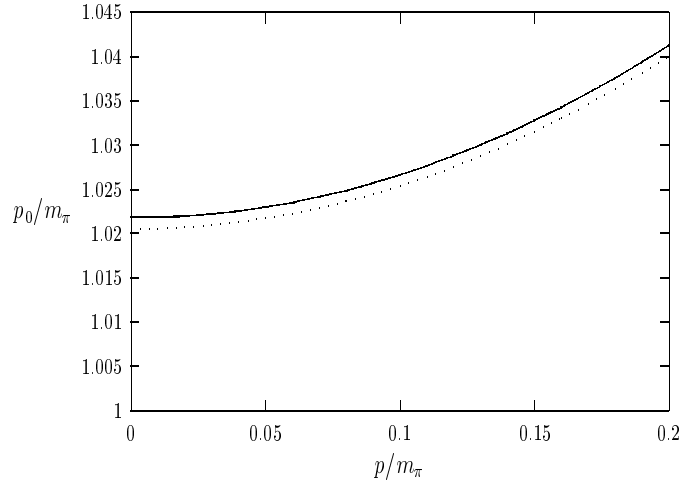


FIG. 4. Pion dispersion relation obtained as the solution to Eq. (23) for $T = m_\pi$ (upper curve). Shown is also the dispersion relation obtained by ignoring the term $\tilde{S}(p_0, p)$ (lower curve).

Figure 4 shows the dispersion relation obtained from Eq. (23) for $T = m_\pi$ where we also display the

solution without the term $\xi^2 \tilde{S}(p_0, p)$. As mentioned before, inclusion of this last term does not alter the shape of the dispersion relation in this kinematical regime.

In conclusion, we have shown that in the linear sigma model at finite temperature, the one-loop modification of the sigma propagator induces a modification in the one-sigma two-pion and four-pion vertices in such a way as to preserve the χ WIs. Strictly speaking, chiral Ward identities fully constrain vertices only in the limit where all the external momenta are zero [6]. Nevertheless, we have also checked that these functions arise from considering all of the possible contributions to the vertices of interest, when maintaining only the zeroth order terms in an expansion in the parameter $(m_\pi/m_\sigma)^2$ [13]. We have used these objects to compute the next to leading order correction to the pion propagator in a pion medium for small momentum and for $T \sim m_\pi$. We have shown that the linear sigma model yields the same result as χ PT at leading order in the parameter $\xi = m_\pi^2/4\pi^2 f_\pi^2$ when use is made of a systematic expansion in the parameter $(m_\pi/m_\sigma)^2$ at zeroth order. This result was to be expected since the kinematical regime we consider is that where the temperature, the pion momentum and the pion mass are treated as small quantities, such as in the case of χ PT. The main modification to the pion dispersion curve in the considered kinematical regime is a perturbative increase of the in-medium pion mass. The shape of the curve is not significantly altered. For a sigma meson with a mass on the order of 600 MeV and in the mentioned kinematical regime, corrections in powers of the parameter $(m_\pi/m_\sigma)^2$ are small. If a lighter sigma meson exists, corrections of order $(m_\pi/m_\sigma)^2$ could be important. It remains to include possible effects introduced by a high nucleonic density. Finally, it is interesting to note that the formalism thus developed could be employed to explore the behavior of the pion dispersion curve in the large momentum region where, in principle, the lowest order χ PT Lagrangian cannot be used. These issues will be treated in a following up work.

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